Written Exam for the M.Sc. in Economics, Winter 2010/2011

## Tax Policy

Final Exam/ Elective Course/ Master’s Course
7 January 2011
(3-hour closed book exam)

## Exercise 1:

Consider the Harberger model of tax incidence. Explain verbally the interplay of the output effect and the substitution effect in determining the incidence of a tax on capital which is only levied on the use of capital in one of the two sectors of the economy.

## Exercise 2:

Assume that preferences of the representative household are given by $U\left(c_{1}, \ldots, c_{N}, l\right)$ where $c_{i}$ denotes the consumption level of commodity $i(i=1, . ., N)$ and $l$ denotes labor supply. The household budget constraint is $p_{1} c_{1}+\ldots+p_{N} c_{N}=w l . p_{i}$ is the consumer price of commodity $i$ and $w$ is the wage rate. The consumer price is $p_{i}=1+t_{i}$. That is, the consumer price consists of the producer price ( $=1$ ) and the tax rate $t_{i}$ levied on commodity $i$. The household chooses consumption levels and labor supply so as to maximize utility.
The government has to choose the commodity tax rates so as to generate tax revenues at a fixed amount $R$.
Different to the standard Ramsey problem, assume that the consumption of commodity $N$ generates a negative externality as captured by the damage term $d\left(c_{N}\right)>0$ which is rising in the consumption of commodity $N$, i.e. $d^{\prime}\left(c_{N}\right)>0$. Total household utility is $U\left(c_{1}, \ldots, c_{N}, l\right)-d\left(c_{N}\right)$. The damage term is neglected by households when choosing their consumption levels and labor supply.
2.1. Set up the decision problem of the representative household. Characterize the consumption choices and labor supply by means of the first order conditions.
2.2. Formally show the effect of a higher tax rate on household utility once household choices are optimized.
2.3. Now, set up the decision problem of the government and characterize the optimal tax rates by means of the first-order conditions. Explain.
2.4. Characterize the optimal tax rates on commodities. In particular, relate them to the optimal rates in the absence of the externality. Explain your finding.

## Exercise 3:

Consider the optimal marginal income tax rate in the Mirrlees model (with only intensive labor supply responses) which a Rawlsian government chooses. Expressed in terms of productivity, the optimal marginal tax rate for productivity level $w$ is characterized by

$$
\frac{T^{\prime}(w)}{1-T^{\prime}(w)}=\frac{1+\varepsilon(w)}{\varepsilon(w)} \frac{1-F(w)}{w f(w)} .
$$

Assume that $\varepsilon(w)=\bar{\varepsilon}$ (that is, it is independent of productivity) and that the productivity follows a Pareto distribution on $[1, \infty)$ :

$$
f(w)=\frac{a}{w^{1+a}} \text { and } F(w)=1-\left(\frac{1}{w}\right)^{a}, a>0 .
$$

### 3.1. Compute the marginal tax rate $T^{\prime}(w)$ for all productivities.

3.2. Have a look at the following figure. Which value of the parameter $a$ of the Pareto distribution is most reasonable for high incomes? Assume for simplicity that the parameter $a$ is the same for the income distribution (depicted in the figure which is taken from Saez, 2001) and the productivity distribution.

3.3. Now, consider a value $\bar{\varepsilon}=0.2$. Compute the marginal income tax rate on high incomes.

## Exercise 4:

Consider the Chamley model of capital taxation. The economy consists of a representative household with preferences

$$
V_{t}=\sum_{\tau=0}^{\infty} \delta^{\tau} u\left(C_{t+\tau}, L_{t+\tau}\right)
$$

where $C_{t+\tau}$ and $L_{t+\tau}$ denote consumption and labor supply in period $t+\tau$. Markets are competitive and, as usual, the household's optimal choice of consumption between two successive periods follows from the standard Euler condition:

$$
\frac{\partial u}{\partial C_{t}}=\delta\left(1+\bar{r}_{t}\right) \frac{\partial u}{\partial C_{t+1}}
$$

where $\bar{r}_{t}$ is the net-of-tax interest rate in period $t$. In each period $t$, output is produced using the constant return to scale production function $F_{t}\left(K_{t}, L_{t}\right)$ where $K_{t}$ denotes the capital stock in period $t$. Capital does not depreciate in production. Hence the resource constraint of the economy is

$$
F_{t}\left(K_{t}, L_{t}\right)+K_{t}=C_{t}+G_{t}+K_{t+1}
$$

$G_{t}$ is government consumption in period $t$. The government can issue public debt and levies a linear tax rate on capital and labor. Thus, the net-of-tax wage rate and interest rate the household receives are $\bar{r}_{t}=\left(1-\tau^{k}\right) r_{t}$ and $\bar{w}_{t}=\left(1-\tau^{l}\right) w_{t}$. The intertemporal budget constraint of the government is

$$
b_{t+1}=\left(1+\bar{r}_{t}\right) b_{t}+\bar{r}_{t} K_{t}+\bar{w}_{t} L_{t}-F\left(K_{t}, L_{t}\right)+G_{t} .
$$

3.1. Write down government tax revenues $R_{t}$ in period $t$.
3.2. Show that tax revenues can be written as

$$
R_{t}=\left(r_{t}-\bar{r}_{t}\right) K_{t}+\left(w_{t}-\bar{w}_{t}\right) L_{t} .
$$

3.3. Write the Lagrangian of the government's decision problem.
3.4. Given the fiscal instruments at hand, the government effectively chooses the capital stock of the economy. Write down the first-order condition for the capital stock $K_{t+1}$.
3.5. Evaluate the first-order condition on a stationary trajectory.
3.6. Determine the optimal difference $r_{t}-\bar{r}_{t}$. Provide an economic intuition for your finding.

